1. General

aNySIM can be used to analyse the dynamic behaviour of a turret or spread moored vessel subject to wind, waves and current. The program predicts the mooring loads and tanker motions when the system is exposed to survival or operational environmental conditions. Optionally the dynamic behaviour of the mooring system can be computed in conjunction with the first and second order motions of the vessel.

Two approaches are possible to model a mooring system in aNySim:

- the quasi-static approach: the shape and tension of mooring lines are derived from catenary formulations. The catenary formulations account for the weight and the axial stiffness of the lines, they do not account for the inertia, the current and the wave forces. The mooring lines are described in 2D, in a vertical plane through anchor and fairlead.
- the dynamic approach: each mooring line is modelled as a chain of discrete masses connected by springs, allowing for inertia, bending stiffness, drag forces and seabed friction to act on individual masses. The mooring lines are described in 3D.

For every mooring system, the user can choose between the static approach and the dynamic approach. If the dynamic approach is chosen then the initial shapes of the lines, which are input for the dynamic model, are still calculated thanks to the static approach.

The number of mooring systems and the number of lines per mooring system is unlimited, although only one mooring system per vessel is possible.

It is not possible to define mooring lines between vessels. The mooring module assumes that each mooring line runs from a fixed point (anchor) to a fairlead of a vessel. It is also assumed that the mooring lines are not branched.

The mooring module offers four different ways of defining the initial state of a mooring system:

1. the line top tension and line azimuth angle are known; the anchor position and line top angle are calculated;
2. the line top angle and line azimuth angle are known; the anchor position and line top tension are calculated;
3. the anchor position is known; the top tension and line top angle are calculated;
4. the anchor radius and line azimuth angle are known; the top tension and line top angle are calculated.
2. Static approach

The static approach uses an iteration procedure within another iteration procedure to determine the mooring line shape and tension for a given anchor and fairlead position:

- an inner iteration procedure: in this procedure the line length or the line angle at the seabed is varied until the height of the line top equals the height of the fairlead for a given horizontal tension;
- an outer iteration procedure: in this procedure the horizontal tension is varied until the horizontal line span equals the distance between the anchor and the fairlead.

During the iteration process extra segment ends are generated at the touch-down point on the seabed and at the intersection with the water surface to derive the correct shape and tension.

The following tolerances are applied for the iteration procedures:

- line tension: 0.0001 kN;
- line top angle: 0.05°;
- vertical line top position: 0.0005 m;
- horizontal line span: 0.0005 m.

The catenary formulations are implemented in the inner iteration procedure. A mooring line may consist of one or more segments. Typically, the division of a mooring line into segments is driven by discontinuities such as different line properties (chain, wire, diameter, rope material, etc.) and the presence of clump weights or buoys.

The analysis starts with the segment which is connected to the anchor. Given the seabed slope, segment length, weight and stiffness and the horizontal tension (set by the outer iteration procedure) it can be determined whether the segment is completely on the seabed, partly lifted from the seabed or completely free from the seabed.

When it rests completely on the seabed the segment top tension and position follow from:

\[
L = L_0 + \frac{T_1 L_0}{EA} - \frac{w L_0^2 \sin(\beta)}{2 EA}
\]

\[
X = L \cos(\beta)
\]

\[
Z = L \sin(\beta)
\]

\[
T_2 = T_1 - w L_0 \sin(\beta)
\]

with:
- segment weight per meter
- segment unstretched length
- segment stretched length
- segment axial stiffness, linearized for \(T_1\)
- tension at segment start
- tension at segment top
\[ \beta \] seabed slope in vertical plane thru mooring line
\[ X \] horizontal segment span
\[ Z \] vertical segment span

When the segment is completely lifted from the seabed its top tension and position follow from:

\[ T_{2,v} = T_{1,v} + L_0 \]
\[ T_1 = \sqrt{T_{1,v}^2 + T_h^2} \]
\[ T_2 = \sqrt{T_{2,v}^2 + T_h^2} \]
\[ Q = 1 + \frac{T_{2,v}T_2 - T_{1,v}T_1 + T_h^2 \ln \left( \frac{T_{2,v} + T_2}{T_{1,v} + T_1} \right)}{2 EA w L_0} \]
\[ X = \frac{Q T_1 \ln \left( \frac{T_{2,v} + T_2}{T_{1,v} + T_1} \right)}{w} \]
\[ Z = \frac{Q (T_2 - T_1)}{w} \]

with:
\[ T_{1,v} \] vertical tension at segment start
\[ T_{2,v} \] vertical tension at segment top
\[ T_h \] horizontal tension (is constant over segment length)

When the segment is partly lifted from the seabed it is split in two: a segment completely resting on the seabed and a segment completely lifted from the seabed. The determination of the touch-down point is done at each timestep of the simulation. The same holds for segments which run through the water surface. These segments are also split in two: a segment completely submerged and a segment completely in air. In this way the correct weight (submerged / in air) can be applied. The intersection point is also determined at each timestep. The water surface level is the still water level: wave elevations are not accounted for.

When the vessel moves towards an anchor it may happen that the mooring line becomes slack: it runs vertical from the fairlead to the seabed. The line takes the shape of an “L” instead of a catenary shape. The catenary formulations cannot be used anymore. To avoid numerical problems the static approach applies the catenary formulations until the line top angle is 88°. Beyond this limit the tension is found by linear interpolation between the tension at an angle of 88° and the tension of the vertical line (90°). The tension in the latter case follows from the weight summation of the segments between seabed and fairlead.

Clump weights and buoys can be attached to the mooring lines. Clump weights have a spherical shape, while buoys have either a spherical or a cylindrical shape. The cylindrical buoys may be horizontal or vertical. At each timestep of the simulation the draft of both clump weights and buoys are calculated to derive the correct nett force which they exert on a mooring line.
In the static approach five synthetic ropes can be used:

- polypropylene / polyester;
- nylon – double braided;
- nylon – 3 strand;
- nylon – 8 strand;
- Dyneema.

The following figures shows the elongation versus the load as percentage of the break load of these ropes:
During a simulation each segment is checked whether the tension exceeds its breaking strength. If so, the line breaks, unless the user has specified to ignore the breaking strength. This option may be useful to derive statistics of the tensions.

In the quasi-static approach the following forces on the mooring lines are ignored:
- The current drag forces.
- The wave forces.
- The bottom friction forces.

3. Dynamic approach

3.1. Basic assumptions

The main assumptions and approximations underlying the simulation algorithm may be summarized as follows:
- All external forces and internal reactions may be lumped to a finite number of nodes.
- Fluid forces may be described by the given empirical relations using relative motions and constant shape coefficients. The local wave velocities at every node is not taken into account in this algorithm.
- Sea floor contact can be simulated by critically damped springs in vertical direction while the horizontal forces are simulated by means of a Coulomb type of friction.

3.2. Equations of motion

The mathematical model for the simulation of the three dimensional behaviour of the mooring lines is based on a lumped mass method. The space wise discretization of the line is obtained by lumping the mass and all forces to a finite number of nodes.

To derive the governing equations of motions for the j-th lumped mass, Newtons law is written in global co-ordinates.

\[
\left( \begin{bmatrix} \mathbf{A}_j \\ \mathbf{a}_j(\tau) \end{bmatrix} \right) \ddot{\mathbf{x}}(\tau) = \mathbf{F}_j(\tau)
\]
where:
\[
\begin{align*}
[A_j] & = \text{inertia matrix for node } j \\
[a_j] & = \text{time dependent added inertia matrix for node } j \\
\tau & = \text{time} \\
\dot{x}_j & = \text{acceleration vector } (x, y, z) \text{ for the } j\text{-th lumped mass} \\
F_{j} & = \text{nodal force vector for node } j
\end{align*}
\]

The added inertia matrix can be derived from the normal and tangential fluid inertia coefficients by directional transformations:
\[
[A_j]\left(\tau\right) = [a_{nj}]\left(\tau\right) + [a_{tj}]\left(\tau\right)
\]

where \(a_{nj}\) and \(a_{tj}\) represent the normal and tangential added mass:
\[
\begin{align*}
a_{nj} & = \rho(C_\text{in} - 1) \frac{\pi}{4} D_\ell^2 L_j \\
a_{tj} & = \rho(C_\text{in} - 1) \frac{\pi}{4} D_\ell^2 L_j
\end{align*}
\]

The matrices \([A_j]\), \([a_j]\), \([\wedge nj]\) and \([\wedge t]\) are given in Appendix L.

The nodal force vector \(F_{j}\) contains the following contributions:

a. segment tension \(T(\tau)\)
b. buoyancy and weight \(F_w\)
c. fluid forces \(F_f(\tau)\)
d. sea floor reactive forces \(F_r(\tau)\)
e. buoy forces \(F_b(\tau)\)

Since the tangential stiffness of the line, represented by its modulus of elasticity \(EA\), is an order of magnitude higher than the stiffness in normal direction, the tension is taken into account in the solution procedure directly.

The tension vector on the \(j\)-th node results from the tension and orientation of the adjacent line segments:
\[
F_T(\tau) = T_j(\tau)\Delta x_j(\tau)/L_j - T_{j-1}(\tau)\Delta x_{j-1}(\tau)/L_{j-1}
\]

where:
\[
\begin{align*}
\Delta x_j & = (x_{j+1} - x_j) \\
L_j & = \text{instantaneous length of segment } j = L_0\left(1 + \frac{T_j(\tau)}{EA_j}\right)
\end{align*}
\]
3.3. Constitutive stress-strain

In order to derive consistent segment tensions and displacements, a Newton-Raphson iteration using the additional constraint equation for the constitutive stress-strain relation is applied:

\[
\begin{align*}
\sigma_j(\tau) &= \left| \Delta x_j(\tau) \right|^2 - L_{oj}^2 \left( 1 + \frac{T_j(\tau)}{EA} \right)^2 \\
T^{k+1}(\tau) &= T^k(\tau) - \left[ \Delta \sigma^k(\tau) \right]^{-1} \sigma^k(\tau)
\end{align*}
\]

where:
\[
\begin{align*}
\sigma &= \text{segment length error vector } (\sigma_1, \ldots, \sigma_j, \ldots, \sigma_N) \\
T^k &= \text{tentative segment tension vector at the } k\text{-th iteration } (T_1, \ldots, T_j, \ldots, T_N) \text{ with } T^k(\tau + \Delta \tau) = T(\tau) \\
[\Delta \sigma] &= \text{length error derivative matrix } \left[ \frac{\partial \sigma}{\partial T} \right]
\end{align*}
\]

Rewriting the constitutive stress-strain relation results in:

\[
\begin{align*}
\sigma_j(\tau) &= L_{oj}^2 \left( 1 - \left( \frac{EA + T_j}{EA + T_{oj}} \right)^2 \right) + 2 \left( x_{oj+1} - x_{oj} \right) \left( \left( \delta x^o_j + 1 \right)^k - \left( \delta x^o_j \right)^k \right) + \left( \left( \delta x^o_j + 1 \right)^k - \left( \delta x^o_j \right)^k \right)^2
\end{align*}
\]

Where \( \left( \delta x^o_j \right)^k \) is the change in position of node \( j \) at time \( n \tau \) at iteration index \( k \).

This formulation is used to prevent loss of accuracy in the calculations.

For each time step the above given system of equations should be solved until acceptable convergence of \( T^k(\tau + \Delta \tau) \) is obtained. The initial tentative tension can be taken equal to the tension in the previous step.

Each node \( j \) is connected to the adjacent nodes \( j-1 \) and \( j+1 \), hence the above given equation represents a tridiagonal \( (N \times 3) \) system. Such equations may be efficiently solved by the so-called Thomas algorithm.

If no acceptable convergence of \( T^k(\tau + \Delta \tau) \) is obtained the algorithm reduces the time step and proceeds from the last time step for which acceptable convergence was obtained. If the time steps becomes to small \( (0.1 / 2^5) \) before convergence, the solution at the previous time step is accepted. This is acceptable because this problem generally only occurs at the slack (less loaded) lines.
3.4. Fluid loads on the mooring lines

This section explains the principles of the calculation of the fluid loads on the mooring lines. Furthermore, the definition of the inertia and drag coefficients, as well as the implementation of the calculation method are presented.

Background

The fluid forces acting on the submerged part of moving slender bodies in waves originate from both the body motions and the water particle motions. The fluid forces can be calculated using the Morison formulation, which is valid for slender bodies. The following formula can be used to calculate the horizontal loads on a slender body (2D condition).

\[ dF = \frac{1}{2} \rho \cdot C_D \cdot \text{AREA} \cdot (u - \dot{x}) \cdot |u - \dot{x}| + \rho \cdot (C_I - 1) \cdot \text{VOLUME} \cdot (\dot{u} - \ddot{x}) + \rho \cdot C_F \cdot \text{VOLUME} \cdot \ddot{x} \]

Drag loads \quad Inertia loads \quad Froude-Krylov force

or, for bodies with a circular cross section (see also pages 224-225 of reference [22]) :

\[ dF = \frac{1}{2} \rho \cdot C_D \cdot D \cdot L \cdot (u - \dot{x}) \cdot |u - \dot{x}| + \rho \cdot C_I \cdot \frac{\pi}{4} \cdot D^2 \cdot L \cdot \ddot{u} - \rho \cdot (C_I - 1) \cdot \frac{\pi}{4} \cdot D^2 \cdot L \cdot \ddot{x} \]

Drag loads \quad Inertia + Froude-Krylov \quad Added mass

In which :

- \( dF \) = horizontal force, [kN]
- \( \rho \) = mass density of water, [tonnes/m\(^2\)]
- \( C_I \) = inertia coefficient, [-]
- \( C_D \) = drag coefficient, [-]
- \( \text{AREA} \) = projected area of element (D.L for cylinders), [m\(^2\)]
- \( \text{VOLUME} \) = volume of element, \((\pi/4).D^2.L\) for cylinders, [m\(^3\)]
- \( u \) = fluid velocity, [m/sec]
- \( \dot{u} \) = fluid acceleration, [m/sec\(^2\)]
- \( \dot{x} \) = body velocity, [m/sec]
- \( \ddot{x} \) = body acceleration, [m/sec\(^2\)]
- \( D \) = cylinder diameter, [m]
- \( L \) = cylinder length, [m]

The above formulation shows 3 different contributions:

Drag forces. Proportional to the relative fluid velocity squared.

Inertia loads (due to fluid accelerations) and a Froude-Krylov force (due to the pressure gradient in the undisturbed wave field). Both are proportional to the fluid accelerations.

Added mass contribution. Proportional to the body accelerations.

The drag forces (contribution 1.) are proportional to the projected area of the mooring line elements, while the inertia and Froude-Krylov loads (contribution 2.), as well as the added mass (contribution 3.) are proportional to the volume of the mooring line element.
The added mass term can be moved to the left hand side of the equation of motions (reaction forces - see Section 5.10.2), while the other two terms remain at the right hand side of the equation (excitation forces).

**Inclined members**
Both the normal and the tangential drag and inertia loads can be determined in a similar manner using $C_I$ and $C_D$ coefficients. The forces are calculated in a local system of axes (see the figure on the next page), using the following equations.

Normal inertia + Froude-Krylov force  
$$F_{in} = \rho \cdot C_{in} \cdot \frac{\pi}{4} \cdot D^2 \cdot L \cdot \dot{u}_n$$

Tangential inertia + Froude-Krylov force  
$$F_{it} = \rho \cdot C_{it} \cdot \frac{\pi}{4} \cdot D^2 \cdot L \cdot \dot{u}_t$$

Normal drag force  
$$F_{Dn} = \frac{1}{2} \cdot \rho \cdot C_{Dn} \cdot D \cdot L \cdot (u_n - \dot{x}_n) \cdot |u_n - \dot{x}_n|$$

Tangential drag force  
$$F_{Dt} = \frac{1}{2} \cdot \rho \cdot C_{Dt} \cdot D \cdot L \cdot (u_t - \dot{x}_t) \cdot |u_t - \dot{x}_t|$$

Normal added mass  
$$a_n = \rho \cdot (C_{in} - 1) \cdot \frac{\pi}{4} \cdot D^2 \cdot L$$

Tangential added mass  
$$a_t = \rho \cdot (C_{it} - 1) \cdot \frac{\pi}{4} \cdot D^2 \cdot L$$

Both for tangential and normal fluid forces, as well as for the added mass, the same reference area (D.L) and volume ($\pi/4.D^2.L$) are used.
Fluid Forces in the local co-ordinate system.

**Note on coefficient definitions**

In the calculation of the inertia and Froude-Krylov loads (contribution 2.), as well as in the calculation of the added mass term (contribution 3.) the coefficient $C_I$ is used. In the inertia + Froude-Krylov term \(C_I D^2\) is used, while in the added mass term \((C_I - 1) D^2\) is used. This difference is a result of the Froude-Krylov contribution, which is present only in the excitation force (contribution 2.) and not in the added mass (contribution 3.). The Froude-Krylov force is by definition proportional to the element volume. Because the $C_I$ coefficient is also based on the element volume, this results in the "-1" difference.

The use of both a \(C_I D^2\)-term and a \((C_I - 1) D^2\)-term in the same equation implies that the $C_I$ coefficients in this equation may only be based on the element volume (or a volumetric equivalent diameter $D$). Using a $C_I$ coefficient based on a different diameter than the volumetric equivalent diameter would lead to a different ratio between "added mass" and "inertia + Froude-Krylov" contributions.

For riser, pipe and wire elements the volumetric equivalent diameter is (almost) equal to the nominal diameter. For chain elements, however, the volumetric equivalent diameter is not equal to the nominal (link) diameter.

**Application**

The orbital velocities and accelerations due to waves are not taken into account in the calculation of fluid loads on the mooring lines. The following formulas are used for the calculation of the drag loads and the added mass.
Normal drag force \[ F_{Dn} = \frac{1}{2} \cdot \rho \cdot C_{Dn} \cdot D \cdot L \cdot (u_n - \dot{x}_n) \cdot |u_n - \dot{x}_n| \]

Tangential drag force \[ F_{Dt} = \frac{1}{2} \cdot \rho \cdot C_{Dt} \cdot D \cdot L \cdot (u_t - \dot{x}_t) \cdot |u_t - \dot{x}_t| \]

Normal added mass \[ a_n = \rho \cdot (C_{In} - 1) \cdot \frac{\pi}{4} \cdot D^2 \cdot L \]

Tangential added mass \[ a_t = \rho \cdot (C_{It} - 1) \cdot \frac{\pi}{4} \cdot D^2 \cdot L \]

The inertia and Froude-Krilov loads on the mooring lines due to the wave orbital motions are not taken into account. This means that the mass coefficient \( C_I \) is only used in the calculation of the added mass. For this reason, it is possible to use mass coefficients \( C_{In} \) and \( C_{It} \) that are based on a nominal diameter for all element types, including chain. This is more convenient, since chain data are generally delivered in terms of nominal (link) diameters.

The nominal diameter based added mass and drag coefficients can be calculated from the volumetric diameter based coefficients (obtained from model tests) using the following formulas.

Diameter : \[ \frac{\pi}{4} \cdot D_{\text{Volumetric}}^2 \cdot L \cdot g = M \cdot L \cdot g \cdot W \cdot L \]

Drag coefficient : \[ C_{D,\text{Nominal}} \cdot D_{\text{Nominal}} = C_{D,\text{Volumetric}} \cdot D_{\text{Volumetric}} \]

\[ C_{D,\text{Nominal}} = C_{D,\text{Volumetric}} \cdot \frac{D_{\text{Volumetric}}}{D_{\text{Nominal}}} \]

Inertia coefficient : \[ (C_{I,\text{Nominal}} - 1) \cdot D_{\text{Nominal}}^2 = (C_{I,\text{Volumetric}} - 1) \cdot D_{\text{Volumetric}}^2 \]

\[ C_{I,\text{Nominal}} = 1 + (C_{I,\text{Volumetric}} - 1) \cdot \frac{D_{\text{Volumetric}}^2}{D_{\text{Nominal}}^2} \]

In which:
\( D_{\text{Volumetric}} \) = equivalent volumetric diameter [mm]
\( D_{\text{Nominal}} \) = nominal diameter [mm]
\( L \) = element length [m]
\( M \) = element mass per meter length [kg/m]
\( W \) = underwater weight per meter length [N/m]
\( C_D \) = drag coefficient [-]
\( C_I \) = mass coefficient [-]
For chain a typical value $D_{volumetric} / D_{nominal} \sim 1.88$ can be used.

The drag and inertia coefficients specified in the input have to be based on the nominal diameter $D$ for all element types (including chain). Since the coefficients $(C_i - 1)$ are used in the calculation of the added mass, the $C_i$-coefficient should be larger than 1.0.

The default values for $C_i$ and $C_D$ are based on model tests for mooring chains, steel wire pipes and riser sections, see Reference [21] and on values from the available literature. For chain, the values based on the equivalent volumetric diameter and based on the nominal diameter have both been included in the table below. In calculations use the values based on the nominal (link) diameter!

The following values are recommended:

<table>
<thead>
<tr>
<th>Material</th>
<th>$C_i$</th>
<th>$C_R$</th>
<th>$C_{Dn}$</th>
<th>$C_{Di}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chain</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Nominal diameter based</td>
<td>3.1</td>
<td>1.7</td>
<td>2.4</td>
<td>0.8</td>
</tr>
<tr>
<td>- Volumetric diameter based</td>
<td>1.6</td>
<td>1.2</td>
<td>1.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Steel wire</td>
<td>1.6</td>
<td>1.2</td>
<td>1.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Pipe / riser</td>
<td>2.0</td>
<td>1.2</td>
<td>1.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Since these coefficients depend on the frequency of oscillation and the cross section of the mooring line, the given values should be considered as average values. For extreme oscillations, special chain links or line components such as clump weights, the fluid reactive force coefficients will be different and should be obtained from model tests.

3.5. Seabed forces

In vertical direction sea floor contact is simulated by means of a linear spring system for each node. Instabilities due to large impacts are prevented by adding a critical damping:

$$F_{ij}(\tau) = \begin{cases} -F_w \left( \frac{z_i(\tau) + z_j(\tau)}{z_{\text{mud}}} \right) \sqrt{\frac{4}{z_{\text{mud}} \cdot g}} & \text{for } z_i < 0 \\ 0 & \text{for } z_i \geq 0 \end{cases}$$

where:
- $z_{\text{mud}}$ = static sea floor deflection
- $g$ = gravitational constant

It should be noted that for the sea floor reactions the sea floor is located at $z = 0$.

In horizontal direction the interaction between line and seafloor is modelled as Coulomb friction:

$$F_{ij}(\tau) = \mu \ N_j \ e_{fr}$$

The friction direction vector $e_{fr}$ follows from the horizontal node velocity component or the horizontal node force component in case the node is in rest.

3.6. Buoys

When the line contains large components when compared with the element volume and fixed in orientation the force formulations as presented in Section 2.2 can not be applied. Therefore an additional direction fixed formulation for fluid forces on such components is optional.

$$F_{bj}(\tau) = \frac{1}{2} \rho \ C_{\text{DA}} \ v_j \ |v_j| + \rho \ C_{\text{IV}} \ v_j$$

where:
- $v_j$ = relative velocity vector in global co-ordinates
- $v_j$ = wave acceleration vector in global co-ordinates
- $C_{\text{DA}}$ = drag coefficient x projected area (vector)
- $C_{\text{IV}}$ = inertia coefficient volume (vector)

The mass (including added mass) of the buoy is added to the mass (see Appendix I) of the concerned node while the weight is taken into account in the node forces.
3.7. Newmark

Assuming that all nodal force contributions are formulated in terms of node positions, velocities and accelerations, the motion equations are solved by a Newmark algorithm which has been adapted for variable time steps.

\[ x_j(\tau + \Delta \tau) = x_j(\tau) - \ddot{x}_j(\tau) \cdot \Delta \tau + \left( \frac{1}{2} - \alpha \right) \dddot{x}_j(\tau) + \alpha \cdot \dddot{x}_j(\tau + \Delta \tau) \cdot \Delta \tau^2 \]

\[ \dddot{x}_j(\tau + \Delta \tau) = \dddot{x}(\tau) + \left\{ (1 - \delta) \dddot{x}_j(\tau) + \delta \cdot \dddot{x}_j(\tau + \Delta \tau) \right\} \cdot \Delta \tau \]

3.8. Discretization aspects

The choice of nodes and elements along the line is of importance from the point of view of accuracy and computational efficiency.

The line discretization should be such that:
- The geometry is represented accurately, the first check can be made on the static configuration.
- Mass and weight lumping is acceptable.
- The angles between two successive elements remain within approximately 20 degrees.
- The fluid forces may be assumed as constant over the element length.

Reference is made to the discussion in Reference [2].

Displacements

- **Keel point:**
  1/2 \( L_{pp} \), midship at keel level.
- **Anchor position:**
  Position of the anchor location given in global coordinate system.
- **Turret location:**
  Position of the origin of the turret coordinate system relative to the vessel keel coordinate system. Per extension, this point is also called mooring system origin (MSO) which can be used for a spread mooring system as well.
- **Fairlead position:**
  Position of the fairlead attachment points given in either the vessel keel coordinate system (spread mooring) or the turret coordinate system (turret moorings).
- **COG:**
  Position of the vessel Centre of Gravity in the vessel keel coordinate system.
4. References

Remove all unquoted references at the end


